1. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m³/min, find the rate at which the water level is rising when the water is 3 m deep. (The volume of a circular cone with radius \( r \) and height \( h \) is given by \( V = \frac{1}{3} \pi r^2 h \)).

\[
\begin{align*}
  r &= 2 \text{ m} \\
  h &= 3 \text{ m} \\
  \frac{dV}{dt} &= 2 \text{ m}^3/\text{min} \\
  \text{Want to find} \quad \frac{dh}{dt} \\
  V &= \frac{1}{3} \pi r^2 h \\
  V &= \frac{1}{3} \pi (\frac{2}{3} h)^2 h \\
  V &= \frac{1}{12} \pi h^3 \\
  \frac{dV}{dt} &= \frac{1}{4} \pi h^2 \frac{dh}{dt} \\
  2 &= \frac{1}{4} \pi (3)^2 \frac{dh}{dt} \\
  \frac{8}{9} \frac{dh}{dt} &= \frac{dh}{dt} \\
  \frac{8}{9} m/\text{min} \text{ or approximately} \quad 0.283 \text{ m/min.}
\end{align*}
\]

2. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure to the right. Let \( h \) be the depth of the coffee in the pot, measured in inches, where \( h \) is a function of time \( t \), measured in seconds. The volume \( V \) of coffee in the pot is changing at the rate of \(-5\pi \sqrt{h}\) cubic inches per second. (The volume \( V \) of a cylinder with radius \( r \) and height \( h \) is \( V = \pi r^2 h \)). Find the rate at which the height is changing at any time \( t \).

\[
\begin{align*}
  r &= 5 \text{ in} \\
  \frac{dV}{dt} &= -5\pi \sqrt{h} \\
  V &= \pi r^2 h \\
  V &= 25\pi h \\
  \frac{dV}{dt} &= 25\pi \frac{dh}{dt} \\
  -5\pi \sqrt{h} &= 25\pi \frac{dh}{dt} \\
  \frac{-\sqrt{h}}{5} &= \frac{dh}{dt} \\
  \text{The rate the height is changing is} \quad \frac{-\sqrt{h}}{5} \text{ in/sec.}
\end{align*}
\]
3. A baseball diamond has the shape of a square with sides 90 feet long. Tweety is just flying around the bases, running from 2nd base (top of the diamond) to third base (left side of diamond) at a speed of 28 feet per second. When Tweety is 30 feet from third base, at what rate is Tweety’s distance from home plate (bottom of diamond) changing?

We know \( \frac{dx}{dt} = -28 \text{ ft/sec} \)

\( x^2 + y^2 = D^2 \) Want to find \( \frac{dD}{dt} \)

When \( x = 30, y = 90, \) so

\[ 30^2 + 90^2 = D^2 \]

\[ 9000 = D^2 \]

\[ 30\sqrt{10} = D \]

When \( x \) is changing, \( y \) is constant.

\[ x^2 + 90^2 = D^2 \] \[ 2(30)(-28) = 2(30\sqrt{10}) \frac{dD}{dt} \]

\[ 2\times \frac{dx}{dt} = 2D \frac{dD}{dt} \]

\[ -880 = -\frac{288}{\sqrt{10}} = \frac{dD}{dt} \]

Tweety’s distance is decreasing at a rate of \( \frac{128}{\sqrt{10}} \) ft/sec, or approximately 8.85 ft/sec.

4. A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius \( r \) of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area \( A \) of the disturbed water changing?

\[ \frac{dr}{dt} = 1 \text{ ft/sec} \]

\[ A = \pi r^2 \]

\[ \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \]

\[ \frac{dA}{dt} = 2\pi (4)(1) \]

\[ \frac{dA}{dt} = 8\pi \]

The area is changing at a rate of \( 8\pi \) ft\(^2\)/sec.
5. A rocket is being launched. It takes off with a velocity of 550 miles per hour. 25 miles away, there is a photographer video-taping the launch. At what rate is the angle of elevation of the camera changing when the rocket achieves an altitude of 25 miles?

\[
\frac{da}{dt} = 550 \text{ mph} \\
\tan \theta = \frac{a}{25} \]

\[
\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{25} \frac{da}{dt} \\
\sec^2 45^\circ \frac{d\theta}{dt} = \frac{1}{25} (550) \\
(\sqrt{2}) \frac{d\theta}{dt} = 22 \Rightarrow \frac{d\theta}{dt} = \frac{22 \sqrt{2}}{2} = 11 \\
\text{The angle is changing at a rate of 11 radians/hr.}
\]

6. A ladder is 25 feet long and leaning against a vertical wall. The bottom of the ladder is pulled horizontally away from the wall at a rate of 3 feet per second. How fast is the top of the ladder sliding down the wall when the bottom is 15 ft. from the wall?

The ladder is sliding down the wall at a rate of \( \frac{3}{4} \) ft/sec or 2.25 ft/sec.
7. A hot air balloon is rising straight up from a level field. It is tracked by a range finder 500ft from the lift off point. Find how fast the balloon is rising at the following moment:

\[ \tan \theta = \frac{y}{500} \]
\[ \sec^2 \theta \frac{dy}{dt} = \frac{1}{500} \frac{dy}{dt} \]
\[ \tan \left( \frac{\pi}{4} \right) (0.14) = \frac{1}{500} \frac{dy}{dt} \]
\[ 140 = \frac{dy}{dt} \]

The balloon is rising at a rate of 140 ft/sec.

8. Air is blown into a spherical balloon at a rate of 36 cubic inches per second. How fast is the radius of the balloon increasing at the instant that the radius is 3 inches?

\[ \frac{dV}{dt} = 36 \text{ in}^3 \]
\[ r = 3 \text{ in} \]
\[ V = \frac{4}{3} \pi r^3 \]
\[ \frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt} \]
\[ 36 = 4 \pi (3)^2 \frac{dr}{dt} \]
\[ \frac{1}{3} = \frac{dr}{dt} \]

The radius is increasing at a rate of \( \frac{1}{3} \) in/sec or approximately 0.333 in/sec.

9. \( x \) and \( y \) are differentiable functions of \( t \) and are related by \( x^3 - 3y = y^2 + \frac{1}{y} + x - 2 \).

Find the rate at which \( x \) is changing when \( x = 2 \) and \( y = 1 \) if \( y \) is decreasing 3 units per second.

\[ \frac{dy}{dt} = -3 \]
\[ x^3 \frac{dx}{dt} + y (3x^2) \frac{dx}{dt} - 3 \frac{dy}{dt} = 2y \frac{dy}{dt} - y^{-2} \frac{dy}{dt} + \frac{dx}{dt} \]
\[ 2^3 (3) + 1 (3x^2) \frac{dx}{dt} - 3(-3) = 2(1)(-3) - 1^{-2} (-3) + \frac{dx}{dt} \]
\[ -24 + 12 \frac{dx}{dt} + 9 = -6 + 3 + \frac{dx}{dt} \]
\[ 11 \frac{dx}{dt} = 12 \]
\[ \frac{dx}{dt} = \frac{12}{11} \]

\( x \) is increasing at a rate of \( \frac{12}{11} \) units/sec.